A New Approach to Flow Problems past a Porous Plate

Osamu Inoue*
Institute of Space and Astronautical Science, Tokyo, Japan

A new method to simulate flows past a two-dimensional porous plate is proposed. The method is a simple application of a discrete vortex method that has been well established for flows past an inclined flat plate. In this method, a flow past a porous plate is described as superposition of the velocity potential of a uniform flow on that of a flow past a flat plate. The effect of porosity is replaced by that of a mass flow rate of the approaching freestream passing through the plate. Calculated flow features are compared with experiments. It is shown that the present method is effective in the simulation of the flow past a porous plate.

Introduction

FLOW past a porous medium like a screen is of practical interest in many engineering problems: for example, removal of dust from jet engine inlet flow, producing turbulence in wind tunnels, smoothing flows, application to parachute problems, etc. Many investigations have been made both experimentally and theoretically. They are reviewed by Laws and Livesey.[1]

In many analytical treatments in the past, a porous plate is replaced by a source distribution.[2-4] For an example, Koo and James[3] devised a mathematical model for steady two-dimensional flow through a screen. In the model, the screen is replaced by a source distribution and the stream function is adjusted to give the correct mass and momentum flow across the screen. The results predict drag with reasonable accuracy so far as the solidity of the screen is small. As demonstrated by the photographs taken by Inoue et al.,[5,6] however, one of the important features of flow past a porous plate is the vortex shedding from the edges of the plate. The vortices shed continuously from the edges tend to dominate the flowfield with the increasing solidity of the plate. Thus, in order to correctly analyze the flowfield past a porous plate, the effect of the vortices shed from the edges should be taken into consideration.

In this paper, a new method to analyze the flowfield past a porous plate is proposed. It is well known that a discrete vortex method is quite effective in simulating a high Reynolds number flow past an inclined flat plate.[7-9] In this method, the effect of vortices shed from the edges of the plate is properly treated. Thus, if we can add the effect of porosity to the flow past a flat plate, then the flow past a porous plate may be adequately analyzed. Our idea is that the effect of porosity can be expressed by superposition of the velocity potential of a uniform flow on the velocity potential of a flow past a flat plate. In the next section, the problem is formulated mathematically and the method of solution is described.

Mathematical Formulation and Numerical Procedure

First, we consider a flow past a flat plate. The fluid is assumed to be set in motion impulsively from rest. This situation can be realized experimentally by moving the plate impulsively in an otherwise stationary fluid. At the initial instance, the flow is assumed to be irrotational and, therefore, the velocities of the flow are infinite at both edges of the plate. In order to remove this divergence, vortices are generated so that they suppress the divergence at the edges (Kutta condition). The viscosity plays its role only at this moment of vortex generation and, except for this moment, the flow is assumed to be inviscid. As in the previous studies,[7-9] the motion of the fluid is followed using conformal mapping. The mapping, which transforms the flat plate of length 2 in the z plane to a circle of radius 1 in the ζ plane, is given by

$$z=(\zeta-1/\zeta)/2 \tag{1}$$

The complex velocity potential f_1 that describes the flow around the flat plate is then given in the ζ plane as follows:

$$f_1=\frac{\zeta+1/\zeta}{2}+i\sum_{k=1}^{N}\frac{\Gamma_k}{2\pi}\log\frac{\zeta-\zeta_k}{\zeta-\zeta_k^*}+i\sum_{l=1}^{N}\frac{\Gamma_l}{2\pi}\log\frac{\zeta-\zeta_l}{\zeta-\zeta_l^*} \tag{2}$$

where the variables are nondimensionalized in terms of the freestream velocity U_∞ and the half-length of the plate. The symbols Γ_k and ζ_k denote the strength and the position of the kth discrete vortex. The subscripts k and l denote the vortices shed from the upper and lower edges of the plate, respectively. The asterisk indicates the image vortices: $\zeta_k^*=1/\bar{\zeta}_k$ and $\zeta_l^*=1/\bar{\zeta}_l$.

Similarly, in terms of Eq. (1), the uniform flow in the z plane, say $f_2=z$, is given in the ζ plane as

$$f_2=(\zeta-1/\zeta)/2 \tag{3}$$

Now, let us consider a flow described by the velocity potential,

$$f_1=\frac{\alpha(\zeta-1/\zeta)}{2}+\frac{(1-\alpha)(\zeta+1/\zeta)}{2}+i\sum_{k=1}^{N}\frac{\Gamma_k}{2\pi}\log\frac{\zeta-\zeta_k}{\zeta-\zeta_k^*}+i\sum_{l=1}^{N}\frac{\Gamma_l}{2\pi}\log\frac{\zeta-\zeta_l}{\zeta-\zeta_l^*} \tag{4}$$

where $\alpha(0\le\alpha\le 1)$ is a constant. As readily seen, the first term of the right-hand side of Eq. (4) represents the contribution due to a uniform flow, the second term the potential flow around a flat plate, and the third and fourth terms generated discrete vortices. For $\alpha=0$, Eq. (4) gives a potential flow around a flat plate, say Eq. (2), while for $\alpha=1$ it gives a uniform flow [Eq. (3)]. For $0<\alpha<1$, a certain amount of mass flow, say α, of the approaching freestream passes through the plate, while the other amount of mass flow, say $(1-\alpha)$, goes around the plate. Therefore, the velocity potential expressed by Eq. (4) is considered to give a flow past a porous plate. It may be noticed that in this method neither the shape nor the size of individual porosity are taken into consideration. In this sense, the applicability of this method may be limited within certain types of porous plates, i.e., screens.

The equation of motion of discrete vortices in the z plane is given by

$$\frac{d\bar{z}_j}{dt}=\left(\frac{df}{dz}\right)_{z=z_j}\qquad (j=k,l) \tag{5}$$

Received Jan. 17, 1985; revision received April 19, 1985.

*Research Associate. Member AIAA.

and, therefore, in the ζ-plane given by

$$\frac{\mathrm{d}\bar{\zeta}_j}{\mathrm{d}t}=\left(\frac{\mathrm{d}f}{\mathrm{d}\zeta}\right)\left|\frac{\mathrm{d}\zeta}{\mathrm{d}z}\right|^2_{\zeta=\zeta_j} \tag{6}$$

The strengths of the vortices Γ_k and Γ_l are determined by using the Kutta condition,

$$\frac{\mathrm{d}f}{\mathrm{d}\zeta}=0 \quad \text{at} \quad \zeta=\pm i \tag{7}$$

The force acting on the plate can be calculated from the generalized Blasius formula,

$$D-iL=i\rho\oint\left(\frac{\partial f}{\partial t}\right)\mathrm{d}\bar{z}+i\frac{\rho}{2}\oint\left(\frac{\mathrm{d}f}{\mathrm{d}z}\right)^2\mathrm{d}z \tag{8}$$

where D is the drag, L the lift, and ρ the density of the fluid.

It should be emphasized that the only difference between the flow past a porous plate treated in this paper and the flow past a flat plate studied in the past[7-9] is the velocity potential [Eq. (4)]. Therefore, we can apply easily to the present problem any one of the numerical methods that have been established for the case of a flat plate. The computational procedure employed in this study is quite similar to that used by Kuwahara.[7] At the initial instance, say $t=0$, two vortices are introduced into the flowfield at two fixed points $\pm z_0$ near the edges of the plate. The strength of each vortex is determined by the Kutta condition [Eq. (7)] with the velocity potential expressed by Eq. (4). These vortices move according to the equation of motion (6) in which the differential quotients $\mathrm{d}\zeta_j/\mathrm{d}t$ are replaced by an appropriate finite difference expression with a time step Δt. At $t=\Delta t$, two new vortices are generated at the points $\pm z_0$. The strengths of the newly generated vortices are again determined by Eq. (7) and vortices move according to Eq. (6). This process repeats at every discrete time. The first-order Euler scheme is used for time integration of Eq. (6), that is,

$$\zeta_j(t+\Delta t)=\zeta_j(t)+\left(\frac{\mathrm{d}\zeta_j}{\mathrm{d}t}\right)\Delta t+\mathcal{O}(\Delta t^2) \tag{9}$$

In this calculation, a variable time step is used and it is determined such that each newly generated vortex does not move beyond a given length Δl. Only a maximum time step $\Delta t_{\max}$ is given beforehand. This method of determination of the time steps makes their values initially very small when the flow pattern changes most rapidly and in the course of time they become nearly constant. The length Δl was prescribed to be $\Delta l=0.02$, 0.05, 0.1, or 0.2. The maximum time step $\Delta t_{\max}$ was 0.1. The positions $\pm z_0$, or equivalently $\pm\zeta_0$, in the ζ plane are prescribed as $\zeta_0=(1+\epsilon)i$. After a number of preliminary calculations, ϵ was set equal to 0.1. In the past studies,[7-9] the discrete vortices in a given vortex cluster were replaced, from the viewpoint of computation time, with an equivalent single vortex whose strength was the sum of the individual strengths and whose position was the center of vorticity of the cluster. This procedure was not employed in this calculation. Therefore, it was possible to examine the flow features during the early stage of flow development in detail, while the fluid motion during the stage of steadily periodic vortex shedding could not be followed sufficiently. It should be also mentioned that any artificial disturbances in order to induce asymmetry of the fluid motion were not introduced into the calculation. The total number of discrete vortices used was about 2300 and the CPU time was about 140 min/case on the FACOM M380. The speed of the FACOM M380 is about twice as great as that of the CDC 7600.

Here the following may be mentioned. The method above described, in which the vortices are generated at two fixed points near the edges of the plate, is known to predict the drag to be about twice as high as the value experimentally obtained during the stage of steadily periodic vortex shedding.[7,9] (Hereafter, this is referred to as the method of fixed points.) Sarpkaya[8] proposed a different method, in which the positions of vortex generation are chosen so as to satisfy the Kutta condition; these move slightly with time. The strength of the generated vortices is determined from the relation $\partial\Gamma/\partial t=U_{\mathrm{sh}}/2$, where U_{sh} is the average velocity in the shear layers. As far as the Strouhal number and the drag coefficient are concerned, the method of Sarpkaya gives better agreement with experiments than the method of fixed points. However, the method of fixed points is much easier to apply than the method of Sarpkaya. Furthermore, according to Kiya and Arie,[9] in an early stage of the flow development, some aspects of vortex pattern around an inclined flat plate showed better agreement with experiments when using the method of fixed points than they did using the method of Sarpkaya. Our main interest here is to study the applicability of the idea expressed by Eq. (4) to flow problems past a porous plate and not to obtain better agreement of the drag with experiment; the method of fixed points is adopted here.

Results and Discussion

Flow past a Flat Plate

Flow features past a flat plate (say, $\alpha=0$) set normal to the freestream have been fairly well studied experimentally.[10,11] Thus, we first compare our calculated results with experiments for the case of a flat plate in order to examine the accuracy of our computational method.

Characteristic flow features during the early stage of flow development are presented in Fig. 1. At the initial stage, the fluid motion is symmetric and a vortex pair is formed. The twin vortices develop with time. With a further increase in time, the twin vortices tend to become asymmetric and at the final stage the Kármán vortex street begins to be formed in the wake.

By using flow visualization techniques, Taneda and Honji[10] studied experimentally the development of the separated flow past a flat plate started impulsively from rest. They investigated the initial symmetric motion of fluid in detail. The flow features during the early stage of flow development calculated in this study are quite similar to those observed by Taneda and Honji. As an example, vortex patterns showing more detail during the early stage of flow development than those in Fig. 1 are presented in Fig. 2. The vortex patterns are quite similar to those obtained in Ref. 10 by means of the electrolysis method. From the streamlines obtained by the aluminum dust method, Taneda and Honji found the following functional relation between the length of symmetric wake bubbles and the dimensionless time:

$$s/d=0.89\tilde{t}^{2/3} \tag{10}$$

where the dimensionless time $\tilde{t}$ is defined as $\tilde{t}=U_\infty t/d$ and d denotes the length of the flat plate. This relation is shown by the solid line in Fig. 3. It is readily seen from Fig. 3 that the agreement between the calculated and the experimental results is reasonably good. After a time equal to approximately 20.0, the flow tends to become asymmetric. It is also seen from Fig. 3 that the effect of Δl, or equivalently the time step, on the flow development is slight.

Freymuth, Bank, and Palmer experimentally studied a uniformly accelerated flow around a symmetric airfoil (NACA 0015) by using flow visualization techniques.[12-14] They obtained many beautiful photographs, one of which (taken at the initial stage of flow development) is present in Fig. 4a. For comparison, one of the calculated results for the case of a flat plate impulsively started from rest is presented in Fig. 4b. Although the flow conditions are rather different from each other, the resemblance between them is apparent.

Regarding the flow features in the transition process from symmetric twin vortices to the asymmetric Kármán vortex street, very few experimental studies have been performed un-

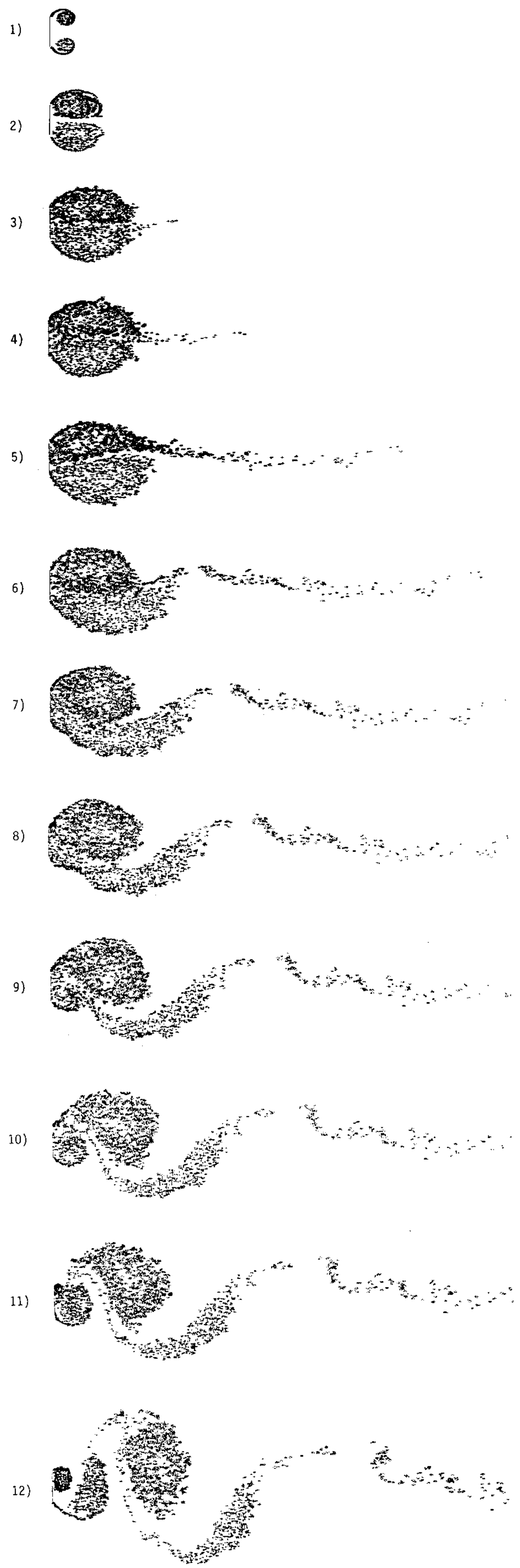


Fig. 1 Flow development around a flat plate impulsively started from rest (streaklines of discrete vortices, $\alpha = 0.0$, $\Delta l = 0.05$).

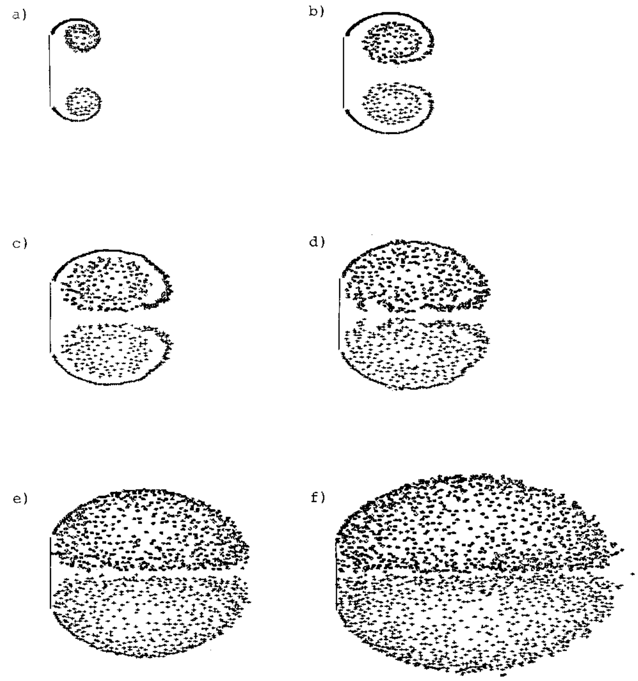


Fig. 2 Vortex patterns at the initial stage of flow development (streaklines of discrete vortices, $\alpha = 0.0$, $\Delta l = 0.1$): a) $\tilde{t} = 0.8$; b) $\tilde{t} = 2.1$; c) $\tilde{t} = 3.6$; d) $\tilde{t} = 5.5$; e) $\tilde{t} = 9.9$; f) $\tilde{t} = 20.2$.

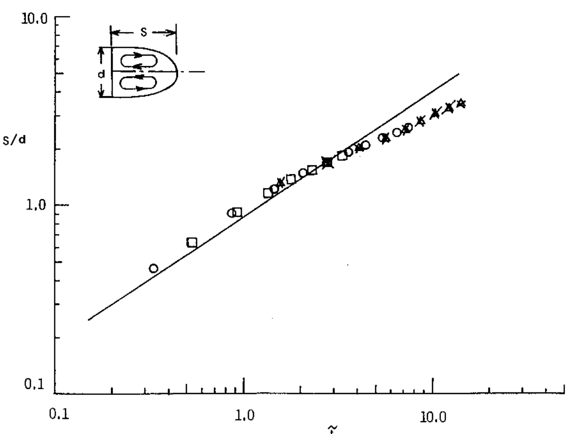
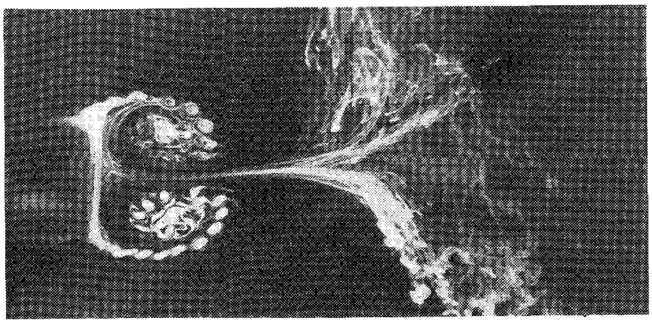


Fig. 3 Vortex length as a function of time for an impulsively started flat plate (from Ref. 10): □ —$\Delta l = 0.2$; ○ —$\Delta l = 0.05$; ✕▲ —$\Delta l = 0.1$.

til now; it is thus rather difficult to compare the calculated results with experiments directly. It may be worth noticing, however, that the appearance of vortices at the start of asymmetry, observed by Freymuth et al.,[13,14] is quite similar to that calculated in this study, as shown in Fig. 5. Figure 5a was taken for the airfoil with the angle of attack 80 deg, while the calculated result in Fig. 5b was obtained for the flat plate with the angle of attack 90 deg (Fig. 1, no. 7). In spite of the difference of the flow conditions between the two cases, the resemblance of flow features at the start of asymmetry is striking.

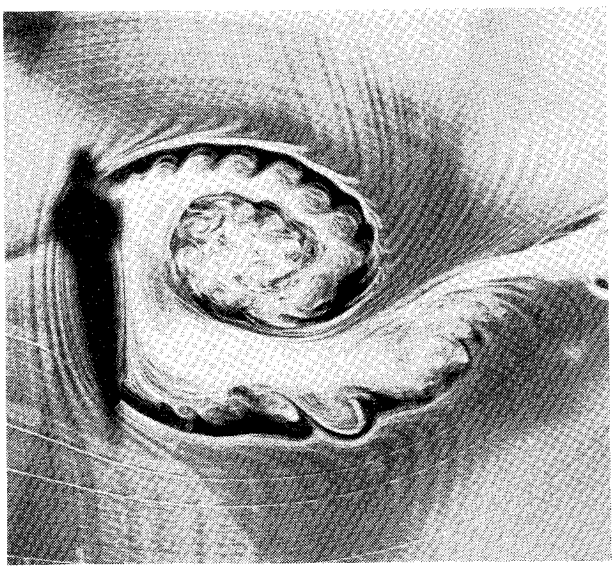
It is of special interest to note that, as seen from Fig. 1, the vortex tearing (or "vortex nipping," as it is called by Freymuth et al.[15]) occurs in the transition process from symmetric twin vortices to the asymmetric Kármán vortex street. This phenomenon is more clearly demonstrated in Fig. 6 where the streaklines of the discrete vortices are depicted. The vortex tearing has been originally predicted by Moore and Saffman[16] and Christiansen and Zabusky[17] for flows with regions of vorticity of opposite signs. The occurrence of the vortex



(a)

(b)

Fig. 4 Vortex patterns at an initial stage of flow development: a) uniformly accelerated flow around an airfoil, angle of attack = 90 deg (from Ref. 14); b) impulsively started flow around a flat plate, angle of attack 90 deg, $\Delta l = 0.05$.



(a)

(b)

Fig. 5 Start of asymmetry of wake bubbles: a) uniformly accelerated flow around an airfoil, angle of attack 80 deg (from Ref. 13); b) impulsively started flow around a flat plate, angle of attack 90 deg, $\Delta l = 0.05$.

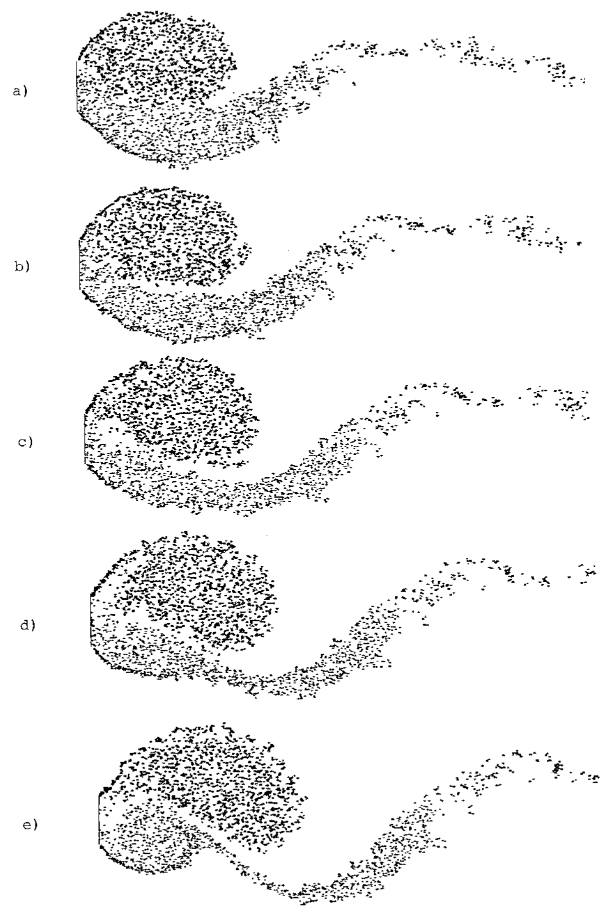


Fig. 6 Vortex patterns in the transition process from symmetric to asymmetric flows (streaklines of discrete vortices, $\alpha = 0.0$, $\Delta l = 0.1$): a) $\tilde{t} = 31.9$; b) $\tilde{t} = 32.9$; c) $\tilde{t} = 33.9$; d) $\tilde{t} = 34.8$; e) $\tilde{t} = 35.7$.

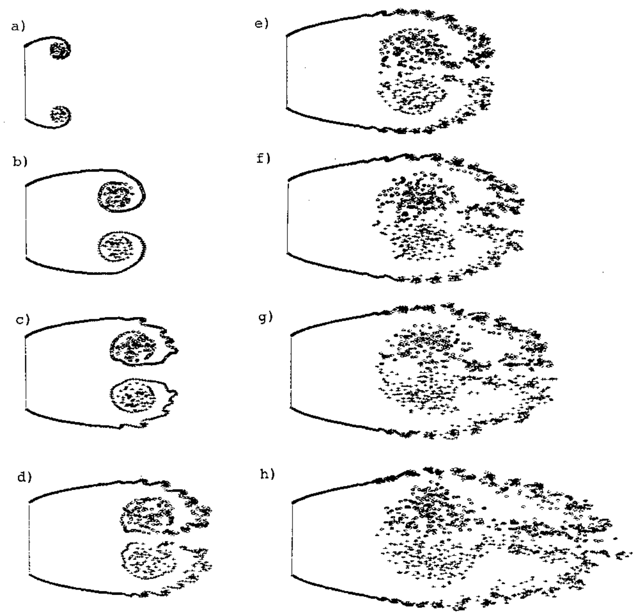


Fig. 7 Vortex patterns at the initial stage of flow development (streaklines of discrete vortices, $\alpha = 0.5$, $\Delta l = 0.05$): a) $\tilde{t} = 0.65$; b) $\tilde{t} = 2.3$; c) $\tilde{t} = 3.1$; d) $\tilde{t} = 4.0$; e) $\tilde{t} = 4.8$; f) $\tilde{t} = 5.7$; g) $\tilde{t} = 6.6$; h) $\tilde{t} = 8.4$.

tearing has recently been confirmed experimentally by Freymuth et al. for a uniformly accelerated flow around an airfoil.[12]

Effect of Porosity

In this study, the effect of porosity of a plate is replaced by that of the mass flow rate α passing through the plate. Characteristic flow features past a porous plate during the early stage of flow development are presented in Fig. 7 for the case of $\alpha = 0.5$. As in the case of a flat plate, the fluid motion at the initial stage is symmetric and a vortex pair is formed. However, the twin vortices are located downstream of the plate, in comparison with the case of a flat plate, because of the flow passing through the plate. With increase in time, the vortices tend to become asymmetric and finally the Kármán vortex street is formed (see Fig. 9 shown later). In the case of a porous plate as well as in the case of a flat plate, the vortex

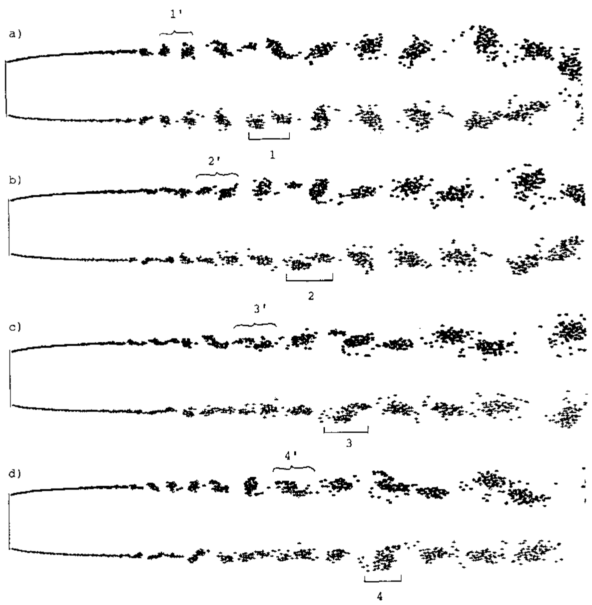


Fig. 8 Vortex pairing in the mixing layers ($\alpha = 0.75$, $\Delta l = 0.02$): a) $\bar{t} = 40.4$; b) $\bar{t} = 42.0$; c) $\bar{t} = 43.7$; d) $\bar{t} = 45.3$.

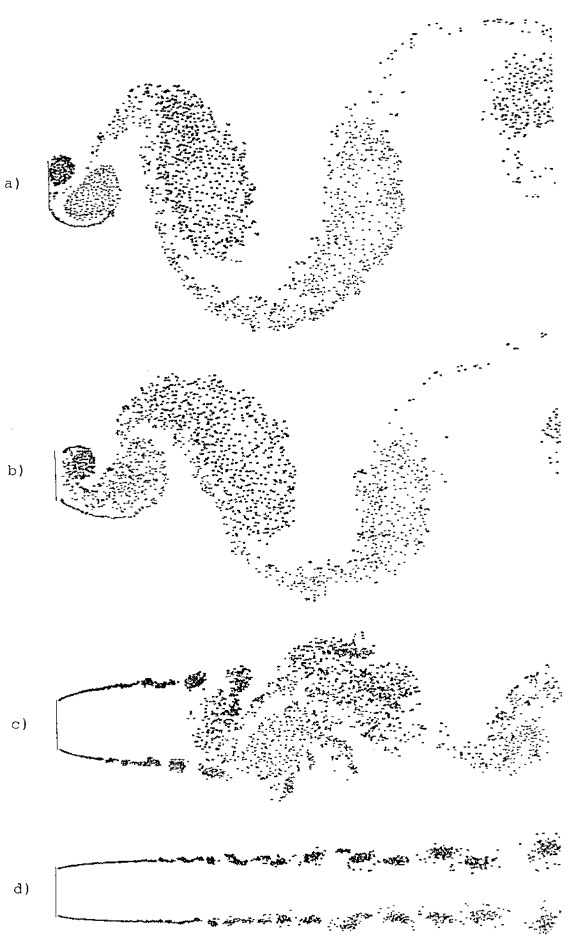


Fig. 9 Streaklines of discrete vortices after fairly long time lapse from start: a) $\alpha = 0.0$, $\bar{t} = 41.1$, $\Delta l = 0.1$; b) $\alpha = 0.25$, $\bar{t} = 36.1$, $\Delta l = 0.1$; c) $\alpha = 0.5$, $\bar{t} = 42.0$, $\Delta l = 0.05$; d) $\alpha = 0.75$, $\bar{t} = 41.9$, $\Delta l = 0.05$.

tearing occurs in the transition process from symmetric twin vortices to the asymmetric Kármán vortex street (for more detail, see Ref. 19). It is of great interest to note from Fig. 7 that due to the Kelvin-Helmholtz instability the discrete vortices shed from the edges of the plate tend to roll up to form separate clusters of vortices, or so-called large eddies, before they enter the wake bubbles; mixing layers are formed and play an important role at the early stage of the wake formation.

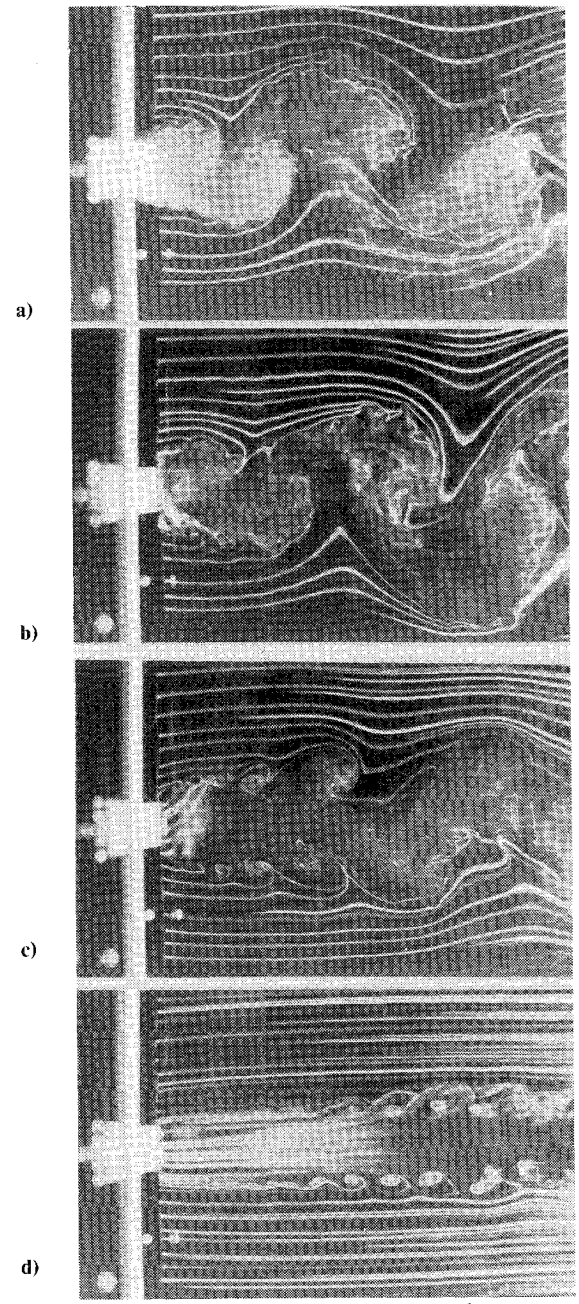


Fig. 10 Flow past a screen (U_∞ 1.6 m/s, $Re = 3 \times 10^3$): a) $\beta = 0.0$ (flat plate); b) $\beta = 0.37$, $M = 200$; c) $\beta = 0.39$, $M = 80$; d) $\beta = 0.55$, $M = 20$.

With the increasing mass flow rate α, the wake bubbles or twin vortices are located further downstream and the mixing layers formed upstream of the wake bubbles develop toward downstream. For example, the vortex patterns for the case of $\alpha = 0.75$ are shown in Fig. 8. We can readily see from Fig. 8 that the vortex pairing process dominates the flowfield in the mixing layers.[6,18] The flow features characterized by the vortex patterns in Fig. 8 are quite similar to those observed experimentally by Inoue et al. for flows past a screen.[5,6] (See also Fig. 10.)

In order to summarize the effect of porosity (or the mass flow rate α) on the flow features past a porous plate, streaklines of discrete vortices after a fairly long time lapse from the start are depicted in Fig. 9 for various mass flow rates. For comparison, presented in Fig. 10 are photographs of the flow past a screen during the stage of steadily periodic vortex shedding, taken by a smoke-wire method of Inoue et al.[19] In Fig. 10, the open-area ratio β is defined as $\beta = (1 - Md/L)^2$, where M is the number of screen mesh, d the diameter of an original wire of the screen, and $L = 25.4$ mm (1 in). It is apparent by comparing Figs. 9 and 10 that the flow

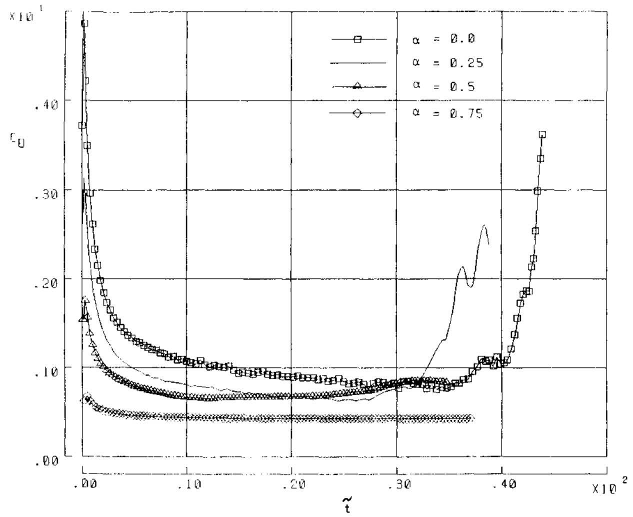


Fig. 11 Drag coefficients as a function of time ($\Delta l = 0.1$).

features calculated by the present method are quite similar to those observed experimentally.

The drag coefficients as a function of time, calculated by using Eq. (8) during the early stage of flow development, are plotted in Fig. 11 for various mass flow rates. For the case of a flat plate, the drag coefficient shows an abrupt increase that initially indicates rapid changes in the flow pattern from an irrotational one to a vortical one. With the development of symmetric twin vortices (Fig. 2), the drag decreases gradually. With further increase in time, flow patterns change from symmetric type to asymmetric Kármán vortex type; the drag increases suddenly and is expected to approach the final oscillating values. This characteristic behavior of the drag has been already shown by Kuwahara[7] for the case of an inclined flat plate with the angle of attack 89 deg. Figure 11 also indicates that the drag decreases with increasing mass flow rate.

Summary and Conclusions

A new method to simulate flows past a porous plate has been proposed. The method is a simple application of the discrete vortex method well established for flows past an inclined flat plate. A flow past a porous plate is described as a superposition of the velocity potential of a uniform flow on that of flow past a flat plate. The effect of porosity is replaced by that of the mass flow rate of the approaching freestream passing through the plate.

The calculated results for the case of a flat plate were compared with the experiments by Taneda and Honji; both the flow visualization results and the results with respect to the length of wake bubbles as a function of time showed a good agreement between calculation and experiment. The flow past a porous plate, calculated for various mass flow rates, was shown to be quite similar to the flow past a screen visualized by a smoke-wire method. The calculated results predict that the vortex tearing (or vortex nipping) occurs in the transition process from symmetric wake bubbles to asymmetric Kármán vortex street.

It is concluded that the present method is very effective to simulate a flow past a porous plate, at least qualitatively. In order to make a more quantitative comparison between the calculated and the experimental results, a relation (or a calibration curve) between the mass flow rate α and the open-area ratio β is required. We are now trying to make the calibration curve and the results will be reported in the future.

Acknowledgments

The author expresses thanks to Prof. K. Kuwahara, ISAS, for his kind advice and useful discussions and to Prof. H. Oguchi, ISAS, for his valuable comments. He also expresses his sincere gratitude to Prof. P. Freymuth, University of Colorado, for the use of his original photographs.

References

[1]Laws, E.M. and Livesey, J.L., "Flow through Screens," *Annual Review of Fluid Mechanics*, Vol. 10, edited by M. Van Dyke, 1978, pp. 247-266.

[2]Taylor, G.I., "Air Resistance of a Flat Plate of Very Porous Material," British Aeronautical Research Council, R&M, 2236, 1944.

[3]Koo, J.K. and James, D.F., "Fluid Flow around and through a Screen," *Journal of Fluid Mechanics*, Vol. 60, 1973, pp. 513-538.

[4]Ito, A., "The Expansion of Flow Due to the Resistance of a Wire Gauze," *Journal of the Japan Society for Aeronautical and Space Sciences*, Vol. 22, No. 247, 1974, pp. 373-377 (in Japanese).

[5]Oguchi, H. and Inoue, O., "Mixing Layer Produced by a Screen and Its Dependence on Initial Conditions," *Journal of Fluid Mechanics*, Vol., 142, 1984, pp. 217-231.

[6]Inoue, O., Sato, S., and Oguchi, H., "Large-Eddy Structures in a Turbulent Mixing Layer," *Journal of the Physical Society of Japan*, Vol. 53, No. 10, 1984, pp. 3378-3384.

[7]Kuwahara, K., "Numerical Study of Flow past an Inclined Flat Plate by an Inviscid Model," *Journal of the Physical Society of Japan*, Vol. 35, No. 5, 1973, pp. 1545-1551.

[8]Sarpkaya, T., "An Inviscid Model of Two-Dimensional Vortex Shedding for Transient and Asymptotically Steady Separated Flow over an Inclined Flat Plate," *Journal of Fluid Mechanics*, Vol. 68, 1975, pp. 109-128.

[9]Kiya, M. and Arie, M., "A Contribution to an Inviscid Vortex-Shedding Model for an Inclined Flat Plate in Uniform Flow," *Journal of Fluid Mechanics*, Vol. 82, 1977, pp. 223-240.

[10]Taneda, S. and Honji, H., "Unsteady Flow past a Flat Plate Normal to the Direction of Motion," *Journal of the Physical Society of Japan*, Vol. 30, No. 1, 1971, pp. 262-272.

[11]Bradbury, L.J.S., "Measurements with a Pulsed-Wire and a Hot-Wire Anemometry in the Highly Turbulent Wake of a Normal Flat Plate," *Journal of Fluid Mechanics*, Vol. 77, 1976, pp. 473-497.

[12]Freymuth, P., Bank, W., and Palmer, M., "First Experimental Evidence of Vortex Splitting," *Physics of Fluids*, Vol. 27, May 1984, pp. 1045-1046.

[13]Freymuth, P., Bank, W., and Palmer, M., "Vortices around Airfoils," *American Scientist*, Vol. 72, 1984, pp. 242-248.

[14]Freymuth, P., Bank, W., and Palmer, M., "Flow Visualization and Hot-Wire Anemometry," *TSI Quarterly*, Vol. IX, No. 4, 1983, pp. 11-14.

[15]Freymuth, P., Bank, W., and Palmer, M., "Further Experimental Evidence of Vortex Splitting," *Journal of Fluid Mechanics*, Vol. 152, 1985, pp. 289-299.

[16]Moore, D.W. and Saffman, P.G., "Structure of a Line Vortex in an Imposed Strain," *Aircraft Wake Turbulence*, edited by J.H. Olsen, A. Goldburg, and M. Rogers, Plenum Press, New York, 1971, p. 569.

[17]Christiansen, J.P. and Zabusky, N.J., "Instability, Coalescence and Fission of Finite Area Vortex Structures," *Journal of Fluid Mechanics*, Vol. 61, 1973, pp. 219-243.

[18]Winant, D. and Browand, F.K., "Vortex Pairing: The Mechanism of Turbulent Mixing Layer Growth at Moderate Reynolds Number," *Journal of Fluid Mechanics*, Vol. 63, 1974, pp. 237-255.

[19]Inoue, O., Ashida, K., and Oguchi, H., "Simulation of a Flow past a Porous Plate," ISAS RN274, 1984.